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A General Thermal-Field Emission Equation

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Abstract: A general emission equation, which contains as its asymptotic limits both the Richardson (thermal emission) and the Fowler Nordheim (field emission) equations, is developed. The general thermal-field equation performs over a range of temperatures, fields, and work function values where the asymptotes are degraded.

Keywords: Field Emission; Thermal Emission; Image Charge Potential; Electron sources, Richardson Equation, Fowler Nordheim Equation

Introduction

Formulae for the evaluation electron emission current density are generally predicated on the high-field low-temperature (Fowler Nordheim Equation) or low-field high-temperature (Richardson-Laue-Dushman Equation) limits of the general emission integral given by

$$J(\beta_F, \beta_T) = \frac{e}{2\pi\hbar} \int_0^\infty D(E_x) f(E_x) dE_x$$

$$f(E_x) = \frac{m}{\pi\beta_T\hbar^2} \ln[1 + \exp(\beta_T(\mu - E_x))] \quad [1]$$

$$D(E_x) \approx C / [1 + \exp(\beta_F(E_o - E_x))]$$

where $D(E)$ is the linearized- θ form of a hyperbolic-tangent approximation to the transmission probability, $f(E)$ is the supply function, β_F and $\beta_T = 1/k_B T$ are slope factors, F [eV/nm] is the product of the field and the electron charge e , μ [eV] is the chemical potential (Fermi level), J [A/cm²] is current density and other terms have their usual meanings. E_x is the energy related to the component of electron momentum directed at the surface – henceforth, the subscript shall be suppressed. E_o is reference point to be determined, and the coefficient C is of order unity (and will be taken henceforth as unity). Eq. [1] can be recast as

$$J(\beta_F, \beta_T) = A_{RLD} T^2 N \left[\frac{\beta_T}{\beta_F}, \beta_F (E_o - \mu), \beta_F E_o \right] \quad [2]$$

$$N(n, s, u) = n \int_{-\infty}^u \ln[1 + e^{n(z-s)}] (1 + e^z)^{-1} dz$$

where $A_{RLD} = 120.17$ A/cm²K². The ratio $n = \beta_T / \beta_F$ is of central importance: its behavior governs the transition from thermal to field emission as n goes from very small to very

large. For all conditions of interest, the large u limit suffices, and is dropped from further consideration. The integral in Eq. [2] can be separated into regions such that series expansions of the components of the integrand can be integrated term-by-term exactly. Collecting terms in the resulting series expansions gives rise to the leading order approximations to $N(n, s)$ of

$$N(n, s) \approx \begin{cases} n^2 e^{-s} \Sigma(1/n) & (n > 1) \\ (n e^{-s} - e^{-ns}) (n-1) & (n \approx 1) \\ e^{-ns} \Sigma(n) & (n < 1) \end{cases} \quad [3]$$

$$\Sigma(x) = \left(1 + \frac{\pi^2}{6} x^2 + \frac{7\pi^4}{360} x^4 \right)$$

The regions $n < 1$ and $n > 1$ shall be referred to as the “thermal” and “field” regimes, respectively. Eq. [2] using Eq. [3] constitutes the *Generalized Thermal-Field model*. The “revised” FN and RLD equations become

$$J_{FN} \Rightarrow A_{RLD} (k_B \beta_F)^{-2} \exp(\beta_F(\mu - E_o)) \Sigma\{\beta_F / \beta_T\} \quad [4]$$

$$J_{RLD} \Rightarrow A_{RLD} (k_B \beta_T)^{-2} \exp(\beta_T(\mu - E_o)) \Sigma\{\beta_T / \beta_F\}$$

where the inherent symmetry between the two limits is manifest and the argument of Σ deserves particular notice. Note, however, that when n is approximately unity, Eq. [4] is no longer adequate and the $n \approx 1$ approximation must be used. The remaining tasks are to determine E_o and β_F .

For a potential barrier described by $V(x)$, the WKB approximation to the “area under the curve” θ is given by

$$\theta(E) = \frac{2\sqrt{2m}}{\hbar} \int_{x_-}^{x_+} (V(x) - E)^{1/2} dx \quad [5]$$

where the limits of the integral are at the zeros of the integrand and $V(x) > E$ otherwise. If we define E_m to be the location of the integrand maximum in Eq. [1], then expressions for E_o and β_F are given by

$$\beta_F = -\partial_E \theta(E = E_m); \quad E_o = E_m + (\theta(E_m) / \beta_F) \quad [6]$$

For $E > V(x)$, a linear extension of $\theta(E)$ is taken: from the parabolic potential WKB approximation we find

$$\theta(E) = \frac{1}{2} \pi \left(\frac{2m}{\hbar^2} \right)^{1/2} Q^{1/4} F^{-3/4} (\mu + \phi - E) \quad [7]$$

where $Q = 0.36$ eV-nm and $\phi = \Phi - \sqrt{4QF}$. In searching for the integral maximum, E_m is an elusive quarry that is best hunted for numerically. For thermal emission conditions, it is found near the barrier maximum, whereas for field emission conditions, it is found near the chemical potential. For Cu-like parameters ($\mu = 7.0$ eV, $\Phi = 4.6$ eV, $T = 800$ K), its behavior is shown in Figure [1].

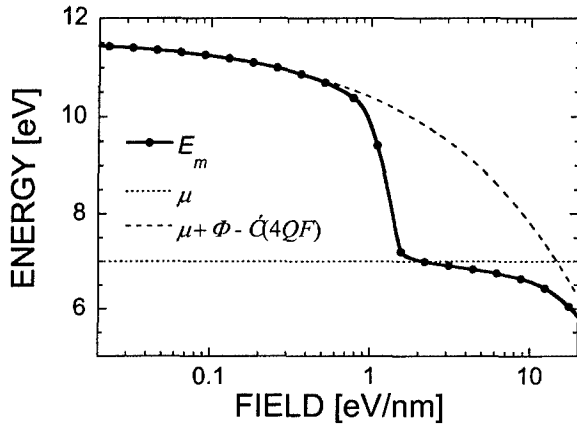


Figure 1 Location of current integrand maximum for copper parameters at 800 Kelvin.

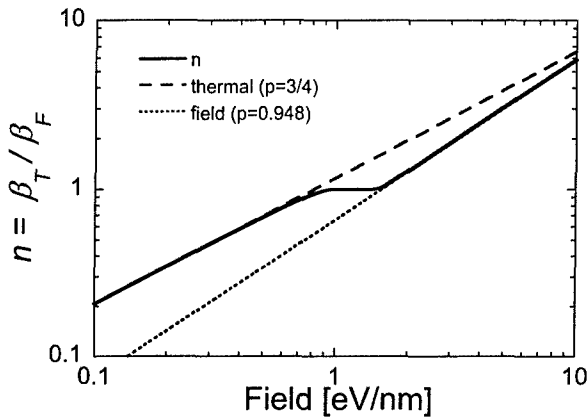


Figure 2 Behavior of the parameter n as a function of field for copper parameters and $T = 800$ K.

Moreover, it has been found that the parameter n satisfies a power-law relation with respect to F at fixed T . Using the

parameters of Figure [1], Figure [2] shows the behavior of $n \propto F^p$. The value of the power p is obtainable through simple models, which shall be taken up separately.

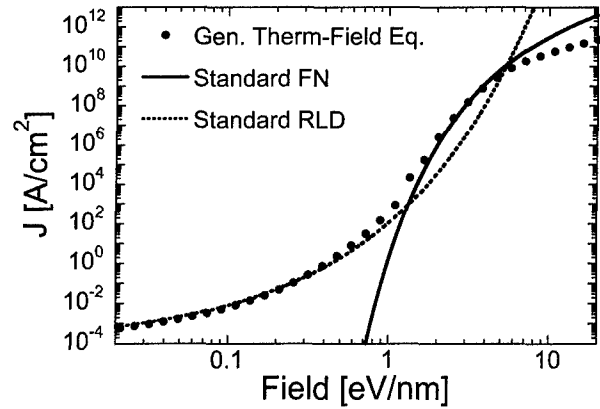


Figure 3 Comparison of Eqs. [3] & [4] to commonly used forms of the FN and RLD equations for copper parameters but for a low Φ of 2.4 eV and $T = 1000$ K.

The performance of the Generalized Thermal-Field equation (GTFE) is shown in comparison to standard implementations of the Fowler Nordheim and Richardson equations but for “challenging” parameters: low work functions at moderate temperatures. The GTFE results are in agreement with numerical evaluation of Eq. [1]. Apart from the expected disagreement in the $n = 1$ region, it is also seen that standard FN result fails at high fields for low work functions, and that standard RLD *overestimates* at low fields because of the impact of E_m ’s shifting location.

In summary, a generalized Thermal-Field emission equation has been derived based on a linear- $\theta(E)$ approximation for the potential evaluated at the maximum of the current integrand. The transition from “thermal-like” emission to “field-like” emission is demarcated by the transition of $n = \beta_T / \beta_F$ from <1 to >1 . from near the barrier maximum to near the Fermi level. The method has general utility for potentials. Revised FN and RLD approximations were given and shown to give good agreement with the exact method.

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